Zeyu Jiang

1. (a)

$(-\infty, 3) \cup (3, +\infty)$

the vertical asymptotes is $x=3$

(b) $x^2-5x+6=0 \Rightarrow x_1=2,\ x_2=3$

$D: (-\infty, 2)(2,3)(3,+\infty)$

the vertical asymptotes ~~is~~ are $x=2$ and $x=3$

(c) $f(x) = \dfrac{3x^2-21x}{6x^2-39x-21} = \dfrac{x(x-7)}{(2x+1)(x-7)} = \dfrac{x}{2x+1}$

$2x+1=0 \qquad x=-\frac{1}{2}$

$D: (-\infty, -\frac{1}{2}), (-\frac{1}{2}, +\infty)$

the vertical asymptotes is $x=-\frac{1}{2}$

(d) $f(x) = \dfrac{x^3+x}{6x^3+x^2-x} = \dfrac{x^2+1}{6x^2+x-1} = \dfrac{x^2+1}{(2x+1)(3x-1)}$

$(2x+1)(3x-1)=0$

$x=-\frac{1}{2},\ x=\frac{1}{3}$

$D: (-\infty, -\frac{1}{2}), (-\frac{1}{2}, \frac{1}{3}), (\frac{1}{3}, +\infty)$

the verticle asymptotes are $x=-\frac{1}{2}$ and $x=\frac{1}{3}$

e). $f(x) = \frac{x^3 - x}{2x^3+9x^2+13x+6} = \frac{x(x^2-1)}{(x+1)(2x+3)(x+2)} = \frac{x(x-1)}{(2x+3)(x+2)}$

D: $(-\infty, -2), (-2, -\frac{3}{2}), (-\frac{3}{2}, +\infty)$

the verticle asymptotes are $x=-2$ and $x=-\frac{3}{2}$

Graph shows that

2. $y = -x^2 + 4$

two holes: $x_1 = -1, x_2 = 2$

So, $y = -x^2+4$ $(x \neq -1$ and $x \neq 2)$

3. (a) $x \to +\infty$ $f(x) = \frac{2x}{x-2} \to 2$ above

$x \to -\infty$ $f(x) = \frac{2x}{x-2} \to 2$ below

So, the horizontal asymptote is $y=2$.

(b) $x \to +\infty$ $g(x) = \frac{-x^2}{x^2+1} \to -1$ above

$x \to -\infty$ $g(x) = \frac{-x^2}{x^2+1} \to -1$ above

So, the horizontal asymptote is $y=-1$

(c) $h(x) = \frac{x-1}{x^2-3x-4}$

$x \to +\infty$, $h(x) \to 0$ above

$x \to -\infty$, $h(x) \to 0$ below.

So, the horizontal asymptote is $y=0$

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4. $\begin{cases} 4b+c=0 \\ \frac{a}{b}=-3 \\ f(\frac{1}{3})=\dfrac{\frac{1}{3}a-2}{\frac{1}{3}b+c}=0 \end{cases}$

$\begin{cases} 4b+c=0 \quad ① \quad c=-4b \quad ④ \\ a=-3b \quad ② \\ \dfrac{\frac{1}{3}a-2}{\frac{1}{3}b+c}=0 \quad ③ \end{cases}$

④ ② $\rightarrow$ ③

$$\frac{\frac{1}{3}\cdot(-3b)-2}{\frac{1}{3}b-4b}=0$$

$$\frac{-b-2}{-\frac{11}{3}b}=0$$

$$-b-2=0$$

$$b=-2$$

set $b=-2$ to $\begin{cases} \frac{a}{b}=-3 \\ 4b+c=0 \end{cases}$

that $\begin{cases} a=6 \\ c=8 \end{cases}$

So. $\begin{cases} a=6 \\ b=-2 \\ c=8 \end{cases}$

5. (a) = (II)

(b) = (V)

(c) = (VI)

(d) = (III)

(e) = (I)

(f) = (IV)

I choose (V) graph, the reason for this is that the (V) graph shows the reflection of x-axis, and the appearance looks like the curvy sea wave.